**Normal forms**

1. A formula is in disjunctive normal form (DNF), if it is written as a disjunction of cubes:

, where  are literals.

1. A formula is in conjunctive normal form (CNF), if it is written as a conjunction of clauses:

, where  are literals.

**Exercise 5**

Transform the formulas  into their equivalent conjunctive and disjunctive normal forms. Using one of these forms prove that  are valid formulas in propositional logic.

1. **U7= (p->(q^r))->((p->q) ^ (p->r))**

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**

**Theoretical results**.

1. To obtain DNF/CNF we apply the normalisation algorithm
   * + - 1. **2. CNF is used to prove validity. If all the clause of CNF(U) are tautologies,**
         2. **then U is valid.**

**3. A clause is a tautology if it contains a pair of opposite literals (p, Ꞁp)**

**U7= (p->(q^r))->((p->q) ^ (p->r)), replace ->, using X->Y ≡Ꞁ X v Y**

**≡ ¬ (p->(q^r)) ∨ ((p->q) ^ (p->r)), replace ->**

**≡ ¬ (¬ p ∨ (q^r)) ∨ ((¬ p ∨ q) ^ (¬ p ∨ r)), de Morgan’s law**

**≡ (p ∧ (¬q ∨¬r)) ∨ ((¬p ∨ q)∧(¬p ∨ r)), distributive laws**

**≡ (p ∧ ¬q) ∨ (p ∧ ¬r) ∨ ¬p ∨ (q∧r) -DNF with 4 cubes**

**≡ (p ∨ p ∨ ¬p ∨ q) ∧ (p ∨ p ∨¬p ∨ r) ∧**

**(p ∨ ¬r ∨¬p ∨ q) ∧ (p ∨ ¬r ∨¬p ∨ r) ∧**

**(¬q ∨ p ∨ ¬p ∨ q) ∧ (¬q ∨ p ∨¬p ∨ r) ∧**

**(¬q ∨ ¬r ∨¬p ∨ q) ∧ (¬q ∨ ¬r ∨¬p ∨ r) -CNF with 8 clauses**

**≡ T V T VT VT VT V T VT VT ≡T**

* + - * 1. **Conclusion: CNF of U7 is true, so U7 is a tautology.**
        2. **Exercise 6**

Using the appropriate normal form write all the models of the following formulas:

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**



**U7=(qVr →p) →(p →r) ˄ q , replace →, using X→Y ≡Ꞁ X v Y**

**≡ Ꞁ (qvr →p) v (p →r) ˄ q , replace →**

**≡ Ꞁ (Ꞁ (qvr) v p) v (Ꞁ p v r) ˄ q**

**≡ ((q v r ) ˄ Ꞁp) v (Ꞁ p v r) ˄ q, apply distributive laws for the 2 subformulas**

**≡ ~~(q ˄ Ꞁp)~~ v (r ˄ Ꞁp) v (Ꞁp ˄ q) v (r ˄ q) DNF with 3 cubes ; U v U = U by the idempotency**

**The cubes provide the models of the formula.**

**Cube: Ꞁp ˄ q provides the models:**

**i1,i2:{p,q,r) -> {T,F}**

**i1(p) = F ; i1(q) = T ; i1(r) = F**

**i2(p) = F ; i2(q) = T ; i2(r) = T**

1. **Cube: r ˄ Ꞁp provides the models:**

**i3,i4:{p,q,r) -> {T,F}**

**i3(p) = F ; i3(q) = F ; i3(r) = T**

**i4(p) = F ; i4(q) = T ; i4(r) = T**

**Cube : r ˄ q**

**i5,i6:{p,q,r) -> {T,F}**

**i5(p) = F ; i5(q) = T ; i5(r) = T**

**i6(p) = F ; i6(q) = T ; i6(r) = T**

**We notice that i2 = i4 = i5.**

**The models of U are i1, i2, i3, i6 (U7)=T**

**i1(U7) = T …....**

**Exercise 7**

Using the appropriate normal form, prove that the following formulas are inconsistent:

**U7= (p->(q->r)) ^ Ꞁ(q->(p->r))**

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**

**Exercise 8**

Write all the anti-models of the following formulas using CNF.

U7= **Ꞁ(Ꞁp v q) v r -> Ꞁp ^ Ꞁ(q^r)**

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**

DNF(U7)=(**Ꞁ** p **˄** not(r)

**Exercise 9**

Using the definition of deduction, prove the following deductions:

**rv(q->p), r v q, Ꞁ  r |-p**

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**

**Exercise 10**

Prove the following theorems using the theorem of deduction and its reverse.

**|- (p->q) ^ (p^q->r) -> (p->r)**

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**

**Exercise 11**

Using the theorem of deduction and its reverse prove that:

|- ( **Ꞁq v p) -> ((s->q) -> (s->p))**

**Logical connectives: V  ˄   Ꞁ   →    ↔**  **↑ ↓ **

**Meta-symbols (express binary semantic relations): |= ≡**

1. **Step1. Apply the reverse of the theorem of deduction**
2. If |- ( **Ꞁq v p) -> ((s->q) -> (s->p)) then**
3. **Then Ꞁq v p** |- **(s->q) -> (s->p))**
4. **Then Ꞁq v p, s->q** |- **s->p**
5. **Then Ꞁq v p, s->q, s** |- **p (\*)**
6. **Step 2: prove deduction (\*)**

**F1: Ꞁq v p ≡ q->p**

**F2: s->q**

**F3: s**

**F3, F2** |-(mp) q:F4

F4, F1 |-(mp) p:F5

1. **Step3: We apply the theorem of deduction**
2. **If Ꞁq v p, s->q, s** |- **p then**
3. **Ꞁq v p, s->q** |- **s->p then**
4. **Ꞁq v p** |- **(s->q) -> (s->p)) then**
5. |- ( **Ꞁq v p) -> ((s->q) -> (s->p))**
6. **Exercise 12**

H1: It is not sunny this afternoon and it is colder than yesterday.

H2: We will go swimming only if it is sunny.

H3: If we do not go swimming, then we will take a canoe trip.

H4: If we take a canoe trip, then we will be home by sunset.

C: We will be home by sunset.

Is C deducible from the set of hypotheses {H1,H2,H3,H4}?

If yes, build its deduction.